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Zero wave resistance for ships moving in shallow channels at supercritical speeds

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This paper deals with the wave pattern and wave resistance of a slender ship moving steadily at supercritical speed in a shallow water channel. Using, successively, linear and nonlinear shallow-water wave theory it is demonstrated that, if the hull form is adapted to speed and channel geometry according to certain rules, the ship waves can be made to form a localized pattern around the ship that moves at the same speed as the ship and at the same time the associated wave resistance can be made to vanish. In the nonlinear case, the zero-wave-resistance ship hull is derived from a KP equation solution of the oblique interaction of two identical solitons. This astonishing phenomenon may be called shallow-channel superconductivity.

1. Introduction

Recently Chen & Sharma (1994, hereinafter abbreviated C-S) investigated the steady asymmetric motion of a slender ship in a straight channel of rectangular cross-section, using nonlinear shallow-water wave theory supported by model tests in a towing tank. It was found by numerical computation and verified by physical measurement that the wave resistance of a ship moving at supercritical speed parallel to the channel axis can be reduced significantly by shifting its track from the channel centreplane to a certain speed-dependent location near one of the channel sidewalls. It was slightly surprising for one might generally expect a symmetric configuration to produce the least resistance and because no comparable effect was noticed in the subcritical speed range. This phenomenon of supercritical wave-resistance reduction by asymmetry is not just an intellectual curiosity but seems to be practically significant. We would like to recapitulate an important finding of C-S that was not explicitly and sufficiently emphasized in the original paper. For a standard ship model of Series 60 hullform of block coefficient 0.6 and length 4.689 m running at depth Froude number 1.3 in a channel of width 9.81 m and water depth 0.5 m, the wave resistance was found to be least when the ship track was 20% of channel width away from the near sidewall. In fact, it was then about 30% lower in both calculations and experiments compared to the symmetric case of ship motion on the channel centreplane, see figure 5(*a*) of C-S.

A brief explanation was also offered in C-S. It was that the bow wave after reflection from the near sidewall eventually hits the afterbody and counteracts the stern wave so that the resultant wave in the wake is weakened. In terms of pressure the reflected bow wave exerts a push on the hull when it acts on the afterbody. Either way, the wave resistance is diminished compared to when bow waves stay clear

(*a*)

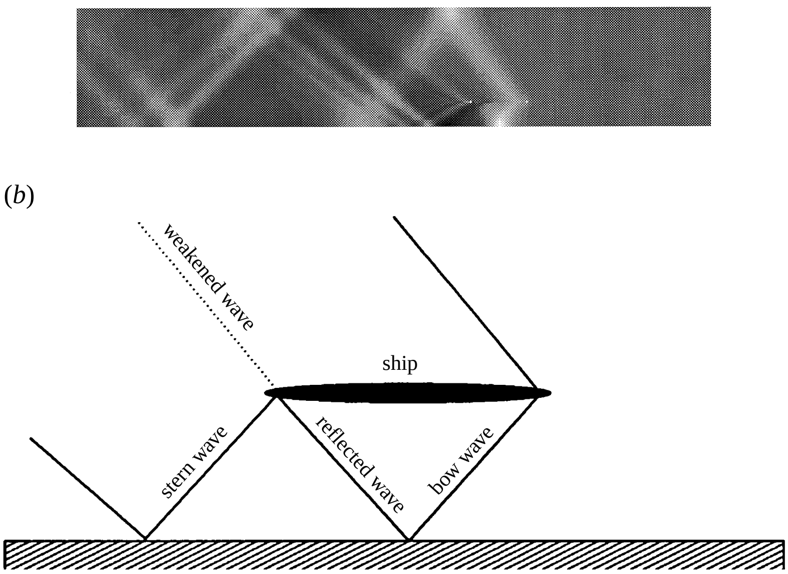

FIGURE 1. (*a*) Calculated wave pattern of a standard ship model of Series 60 hull form in off-centre motion (from left to right) parallel to channel axis at 20% channel width from the near wall at supercritical depth Froude number $U = 1.3$. The wave resistance non-dimensionalized by ship weight $C_w = 0.036$, its reduction $(C_{w_o} - C_w)/C_{w_o} = 30\%$, where C_{w_o} is the wave resistance value for motion along the channel centreline. (*b*) Schematic of the mechanism of wave resistance reduction by sidewall reflection

of the stern. Similar favourable wave interference effects based on classical linear theory have been reported earlier, e.g. for the catamaran configuration (theoretically equivalent to a single sidewall) in deep as well as finite-depth water by Eggers (1955, figures 2 and 18) and for a finite-width channel at supercritical speeds by Kirsch (1966, figures 2–9). The phenomenon is less apparent in the subcritical speed range (and, of course, in deep water) because the relatively strong dispersion prevents an effective interference between bow and stern waves. By contrast, in the supercritical speed range all ship waves have almost the same phase speed and parallel crest lines, only weakly dependent on wavelength, so that the interaction of reflected bow waves with the stern waves becomes very distinct.

In order to visualize the mechanism of wave reduction more clearly, we reran our computer program 'shallowtank' for the minimal wave-resistance case mentioned above and obtained the associated wave pattern, see figure 1. At supercritical speeds the bow and stern waves are dominant and the ship wave pattern is just like the shock wave of an aerofoil in supersonic flight. The bow creates a free-surface elevation; the stern, a depression. It is clearly observed in figure 1 that the starboard bow wave (here on the near-wall side) is reflected by the channel sidewall and in extending across the three-dimensional stern it almost completely cancels the port stern wave (on the far-wall side) as evidenced by comparison with the strong starboard stern wave (on the near-wall side).

Naturally, one is inspired by figure 1 to also move the far wall close to the ship so that the starboard stern wave would be cancelled as well by the reflected port bow wave. By numerical experimentation we obtained the optimal channel width for a

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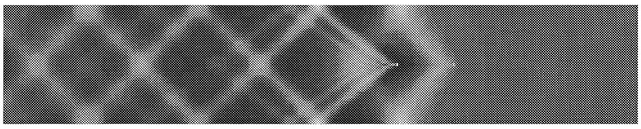

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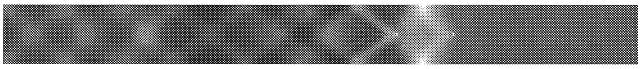

FIGURE 2. Calculated wave patterns of a standard ship model of Series 60 hull form moving along the channel centreline at depth Froude number $U = 1.3$ in (*a*) the original channel of width $w^* = 9.81$ m with specific wave resistance $C_{w_o} = 0.051$ and (*b*) the optimized channel of width $w^* = 4.9$ m with specific wave resistance $C_{w_o} = 0.013$; the achieved wave resistance reduction $(0.051 - 0.013)/0.051 = 75\%$.

symmetric ship–channel configuration at depth Froude number 1.3. Coincidentally, it turned out to be 4.9 m, just half of the original value. The wave resistance is now reduced by 75%. The associated wave patterns in the optimal narrow channel and in the original channel are shown in figure 2. It is obvious that the resultant waves in the ship wake are much weaker in the optimal width channel than in the original. The lower energy density in the optimal channel multiplied by its reduced width explains the dramatic drop in wave resistance.

We were strongly stimulated by this success to seek a full theoretical understanding of this intriguing phenomenon and eventually discovered that the wave resistance of a ship in a channel can be made to vanish within the framework of a linear shallow-water wave approximation and, furthermore, even in a more accurate nonlinear theory. By analogy to electrical conductors, which are known to become superconductive (zero electrical resistance) under certain conditions, we propose the name ‘shallow channel superconductivity’ for this phenomenon.

Here, a few comments on the exciting topic of waveless ships are in order. Naval architects have always striven for ships without waves. Bow bulbs or submerged wings attached to ship hulls are good examples of mutual cancellation of waves originating from different sources. In principle, ship waves can be even completely eliminated within linear theory. Several interesting theoretical proposals have been made recently by Tuck (1989), Tuck & Tulin (1992), and Tulin & Oshri (1994). The experimental investigation of Mori (1993) is also worth mentioning. But to our knowledge the idea of ship superconductivity in shallow water has never been demonstrated or proposed before. Intuitively speaking, since wave dispersion is weaker in shallow water, the interference between waves arising from different origins becomes more effective than in deep water, especially at supercritical speeds. In this sense waveless ships in shallow water may be more feasible.

During revision of this paper we became aware of two important historic papers, namely Russell (1837) and Busemann (1935), that are related to our topic. In his article Russell, the well-known discoverer of the solitary wave, reported that a spirited horse pulling a boat in a canal had drawn the boat up onto its own wave leading to a significant reduction in resistance. The boat owner had noted this and the event lead to ‘high-speed’ services on some of Britain’s canals in the 1820s and ’30s. Although we cannot know the exact mechanism of this phenomenon, we conjecture that the diminished resistance occurred at a slightly supercritical speed where a favourable

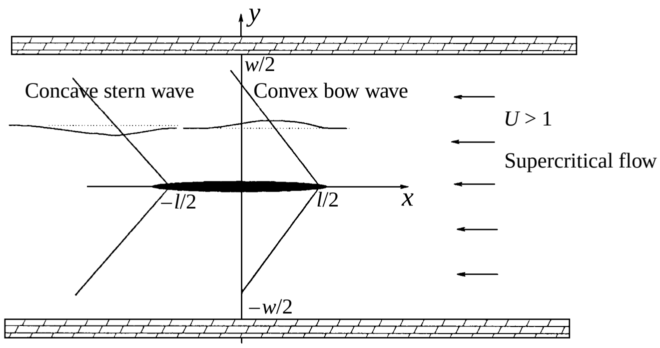

FIGURE 3. Schematic of the problem.

interference between stern waves and bank-reflected bow waves might have taken place as proposed here. The second paper was given at the historic Volta Congress in 1935 and came to our notice through a recent memorial paper by Ferrari (1996) who was an original participant. In that paper Busemann proposed to cancel the wave drag of supersonic aeroplanes, at least at zero lift and in a two-dimensional approximation, by means of a biplane configuration. This would be accomplished by coupling two airfoils in such a way as to form a kind of Venturi tube: the waves produced by each one of the two internal surfaces and incident upon the other would not be reflected. This idea came to be known as Busemann's biplane. Indeed, it is also the basic idea of our paper in view of the analogy between two-dimensional supersonic gas dynamics and shallow-water waves. The essential difference lies in our nonlinear treatment of water waves involving dispersion.

In §2 we illustrate our basic idea with a simple linear two-dimensional wave equation applicable to a supercritical ship in shallow water or to an aerofoil in supersonic flight, neglecting both dispersive and nonlinear effects. We show that the wave resistance acting on a slender body in a channel becomes zero for a suitable combination of body speed, channel depth and channel width if the afterbody geometry is adapted to an arbitrary forebody according to a simple rule. In §3 we extend the analysis by considering a more accurate nonlinear Kadomtsev–Petviashvili (KP) equation (2-D KdV equation). By exploiting its known two-soliton solution we demonstrate that a zero-wave-resistance ship is still possible, albeit with strong restrictions on body geometry. The wave pattern in the zero-wave-resistance condition is localized and characterized by phase lines which form a rhombus both in linear and nonlinear theory. In §4 we conclude the discussion of the superconducting channel and indicate the prospects of a practical catamaran of zero wave resistance in shallow water of any width. Certain mathematical details of the two-soliton solution and the zero-wave-resistance ship hull derived for arbitrary channel width are supplied in the Appendix.

2. Linear analysis

The problem of a slender ship in a shallow channel or a two-dimensional thin aerofoil in a channel is sketched in figure 3. It is well known (Wehausen & Laitone 1960, p. 668) that steady supercritical shallow-water flow around a slender ship and two-dimensional supersonic gas flow around a thin aerofoil are both approximately governed by the same linearized wave equation with depth Froude number in the former playing the role of Mach number in the latter. In the following we discuss only the shallow-water flow but the results are valid analogously also for the supersonic gas flow.

Neglecting the nonlinear and the dispersive effects for the time being, we have the governing equation (Tuck 1966)

$$(1 - U^2)\varphi_{xx} + \varphi_{yy} = 0 \quad (-\infty < x < \infty, \quad -w/2 < y < w/2), \tag{2.1}$$

the boundary condition at the ship location, for symmetric flow,

$$\varphi_y|_{y=\pm 0} = \mp\tfrac{1}{2}U\frac{\mathrm{d}S(x)}{\mathrm{d}x} \quad (-l/2 < x < l/2), \tag{2.2}$$

and the boundary condition on the channel sidewalls

$$\varphi_y|_{y=\pm w/2} = 0 \quad (-\infty < x < \infty), \tag{2.3}$$

where φ is depth-averaged velocity potential, $S(x)$ is distribution of ship cross-sectional area, w is channel width, l is ship length and U is depth Froude number. All variables are non-dimensional, having been derived from corresponding dimensional variables, denoted by asterisks (*), as follows:

$$\left.\begin{array}{l} x = x^*/h^*, \quad y = y^*/h^*, \quad \varphi = \varphi^*/(h^*(g^*h^*)^{1/2}), \quad S = S^*/h^{*2}, \\ w = w^*/h^*, \quad l = l^*/h^*, \quad U = U^*/(g^*h^*)^{1/2}, \end{array}\right\} \tag{2.4}$$

where h^* is water depth and g^* is acceleration due to gravity.

The wave resistance acting on the ship ($R_w = R_w^*/(\rho^*g^*h^{*3})$, where ρ^* is water density) can be obtained exactly by integrating the longitudinal component of pressure on the wetted hull surface S_w, namely

$$R_w = \int_{S_w} p n_x \mathrm{d}s, \tag{2.5}$$

where $p = p^*/(\rho^*g^*h^*)$ is the pressure and n_x is the x-component of the outward normal vector on S_w at any point. The pressure is usually linearized in shallow-water theory as

$$p = U\frac{\partial\varphi}{\partial x} - z. \tag{2.6}$$

Then the linearized approximation of wave resistance becomes

$$R_w = -\int_{-l/2}^{l/2} U\frac{\partial\varphi}{\partial x}\bigg|_{y=0^+} S'(x)\mathrm{d}x, \tag{2.7}$$

where the prime denotes derivative with respect to the independent variable. The free-surface displacement $\zeta = \zeta^*/h^*$ has the linearized approximation

$$\zeta = U\varphi_x. \tag{2.8}$$

Here we consider only supercritical speeds, i.e. $U > 1$. In this case the equation (2.1) is hyperbolic. Since the problem is symmetric about the x-axis, it is sufficient to consider the half-region $y > 0$. The solution of the hyperbolic problem is easily obtained by the characteristic line theory. There exist two characteristic lines:

$$\xi = x + y\cot\alpha = \text{const}, \quad \eta = x - y\cot\alpha = \text{const}, \tag{2.9}$$

yielding the general solution,

$$\varphi(x, y) = F(\xi) + G(\eta), \tag{2.10}$$

where $\tan\alpha = 1/(U^2-1)^{1/2}$ is the slope of the lines. F and G are determined by the boundary conditions and the causality condition. First, let us suppose there are no channel sidewalls, i.e. the ship moves in sidewise-unrestricted shallow water. Here the causality condition implies that any disturbance is propagated only downstream. Hence, the realistic solution has only one branch, i.e. for $y > 0$

$$\varphi(x, y) = F(\xi). \tag{2.11}$$

Substituting the above expression into the ship boundary condition (2.2), we get

$$F(\xi) = -\frac{U}{2(U^2-1)^{1/2}} S(\xi). \tag{2.12}$$

Using the velocity potential given by (2.11) and (2.12), the wave resistance in (2.7) is found to be

$$R_w = \frac{U^2}{2(U^2-1)^{1/2}} \int_{-l/2}^{l/2} S'(x)^2 \mathrm{d}x, \tag{2.13}$$

which is a well-known supercritical result, see equation (6.22) of Tuck (1966). Note that every cross-section of the ship radiates its own wave disturbance. But they are all parallel and do not interact with each other at all since there is no dispersion. The wave resistance given by (2.13) is the sum of their individual positive contributions. The free-surface displacement in (2.8) is also found to be

$$\zeta(x, y) = -\frac{U^2}{2(U^2-1)^{1/2}} S'(\xi) \quad (y > 0). \tag{2.14}$$

For a normal hull form, $S'(x)$ is negative in the forebody $x > 0$ and positive in the afterbody $x < 0$. So the bow generates a wave elevation while the stern generates a wave depression in the supercritical case. This corresponds to a shock wave of compression at the leading edge and of expansion at the trailing edge in supersonic flow. We note in passing that each elementary wave propagates in a direction normal to its crest line at exactly the critical speed of unity and the slope is such that the induced wave speed in the direction of motion of the ship is just equal to ship speed U.

Now we reintroduce the channel sidewalls. On each side the ship-generated waves are obliquely incident on the sidewall and reflected by it. It is obvious that if the reflected bow wave elevation just reaches the stern where the ship itself would create a wave depression, then the two effects will counteract each other and the resulting free-surface displacement will be small or even totally absent if the original bow and stern waves were of exactly equal magnitude. In the following, we analytically look for a geometric condition on the hull form in combination with the principal parameters, i.e. channel width w, ship length l and depth Froude number U, such that the resulting waves in the wake would theoretically disappear and consequently the wave resistance would be zero. By an admittedly superficial analogy to electrical conductors at near zero temperatures this might be called a state of superconductivity.

In the case of zero wave resistance, the characteristic lines run as sketched in figure 4(a). The first wave extends from the ship bow B ($x = l/2, y = 0$) through its reflecting point $\mathrm{B_r}$ ($x = l/2 - (w/2)\cot\alpha, y = w/2$) on the sidewall up to the point B′ ($x = l/2 - w\cot\alpha, y = 0$) somewhere in the afterbody. The last wave extends from the fore shoulder S through its reflecting point $\mathrm{S_r}$ on the sidewall up to the ship stern S′ ($x = -l/2, y = 0$). The intermediate waves extend similarly from arbitrary points P (between B and S) through $\mathrm{P_r}$ up to P′. Complete cancellation requires that the wave

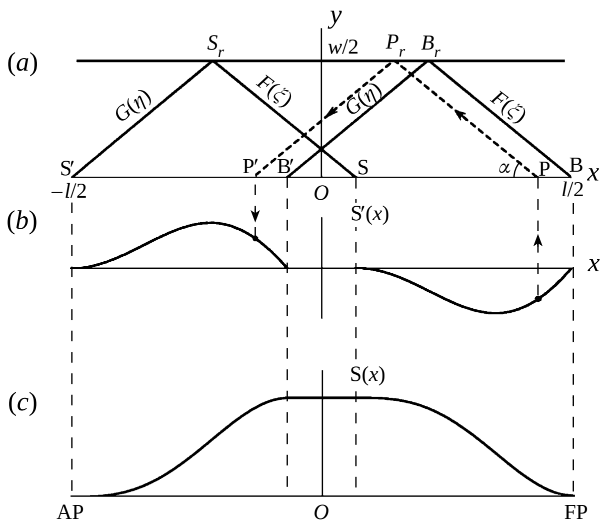

FIGURE 4. Schematic of the zero-wave-resistance solution in linear theory.

elevation primarily generated at the point P must be exactly the same size as the wave depression associated with the point P′. Since the generated wave disturbance is linearly proportional to the slope of the cross-sectional area curve, this requirement means that $S'(x)$ at P′ is of equal magnitude but opposite sign to $S'(x)$ at P, see figure 4(*b*). The geometric condition for the state of zero wave resistance is, therefore,

$$S'(x) = \begin{cases} \text{arbitrary}, & w\cot\alpha - l/2 < x \leqslant l/2 \\ 0, & l/2 - w\cot\alpha \leqslant x \leqslant w\cot\alpha - l/2 \\ -S'(x + w\cot\alpha), & -l/2 \leqslant x \leqslant l/2 - w\cot\alpha, \end{cases} \tag{2.15}$$

along with the minimum length condition $l > w\cot\alpha$. For simplicity, we add a maximum length condition, $l \leqslant 2w\cot\alpha$, which ensures that the point B′ lies in the afterbody. But note that this is dispensable if one is willing to cope with the complexity of multiple reflections. Although mathematically the function $S(x)$ between points B and S can be arbitrary, it is realistic to assume that $S'(x)$ is non-positive in the forebody and that $S(x)$ vanishes at the bow. Moreover, if we want the hull form to be smooth at the shoulders S and B′, the curve $S(x)$ must have cusps at the bow and stern, i.e. $S'(-l/2) = S'(l/2) = 0$. This implies either a pointed bow or a wedge-shaped bow with zero angle of entrance. Such a zero-wave-resistance solution is schematically shown in figure 4. It is obvious that the function $S(x)$ sketched in figure 4(*c*) forms a closed body, not necessarily fore-and-aft symmetric.

We now check analytically whether there are really no waves left in the ship wake under the above conditions. Before the point B′ the ship is not affected by the reflected waves. So the solution along the x-axis in this range is the same as in an infinitely wide channel, i.e.

$$\varphi(x, y) = F(\xi) = -\frac{U}{2(U^2 - 1)^{1/2}} S(\xi) \quad (x > l/2 - w\cot\alpha, \;\; y = +0). \tag{2.16}$$

After the point B′, the solution is generally,

$$\varphi(x, y) = F(\xi) + G(\eta) \quad (x \leqslant l/2 - w\cot\alpha, \;\; y = +0), \tag{2.17}$$

where $F(\xi)$ represents a possible ship-generated wave (so far unknown) and $G(\eta)$ a sidewall-reflected wave (already known). To achieve our purpose we must prove that $F'(\xi) = 0$ along $x \leqslant l/2 - w\cot\alpha$, $y = +0$. Now, $G(\eta)$ at an arbitrary point P′ comes originally from the point P via reflection at P_r, so that by virtue of sidewall condition (2.3) we get

$$G'(x) = F'(x + w\cot\alpha) = -\frac{U}{2(U^2-1)^{1/2}} S'(x + w\cot\alpha) \quad (x \leqslant l/2 - w\cot\alpha,\ \ y = +0). \tag{2.18}$$

Using (2.17), (2.18) and (2.15) we derive the normal velocity at $y = +0$ and finally obtain

$$\frac{\partial\varphi}{\partial y} = F'(x)\cot\alpha - \tfrac{1}{2}U\,S'(x) \quad (x \leqslant l/2 - w\cot\alpha,\ \ y = +0). \tag{2.19}$$

Comparing it with the ship boundary condition (2.2), we directly obtain

$$F'(x) = 0 \quad (x \leqslant l/2 - w\cot\alpha). \tag{2.20}$$

This result means that the waves generated by the ship forebody, after reflection at the channel sidewall, are totally absorbed by the ship afterbody and, consequently, there are absolutely no waves in the ship wake. Zero wave resistance can be verified by evaluating the integral (2.7) in the following manner. The integration is divided into three intervals: B to S, S to B′, and B′ to S′. Since $S'(x) = 0$ in the middle interval S to B′, its integration makes no contribution. Since no wave is incident upon the interval B to S and no wave is radiated from the interval B′ to S′, i.e. (2.16) and (2.17) with (2.20) hold, respectively, the full integral can be written as

$$R_w = -U\left[\int_{-l/2}^{l/2 - w\cot\alpha} G'(x)S'(x)\mathrm{d}x + \int_{w\cot\alpha - l/2}^{l/2} F'(x)S'(x)\mathrm{d}x\right]. \tag{2.21}$$

Further, substituting (2.16) and (2.18) into the above and using (2.15) we get

$$\begin{aligned} R_w &= \frac{U^2}{2(U^2-1)^{1/2}}\left[-\int_{-l/2}^{l/2 - w\cot\alpha} S'(x+w\cot\alpha)S'(x+w\cot\alpha)\mathrm{d}x + \int_{w\cot\alpha - l/2}^{l/2} S'(x)S'(x)\mathrm{d}x\right]. \\ &= 0. \end{aligned} \tag{2.22}$$

This completes the mathematical proof that a ship with cross-sectional area distribution (2.15) and satisfying the length conditions $w/(U^2-1)^{1/2} < l \leqslant 2w/(U^2-1)^{1/2}$ is superconductive in the channel.

To see the superconductive phenomenon in detail we take an example. Let $U^2 = 2$, $l/2 = 1^+$ and $w/2 = 1$, so $\cot\alpha = 1$. The geometric condition (2.15) is satisfied in the simplest case by a rectangular cross-sectional area distribution, i.e.

$$S'(x) = \delta(x+1) - \delta(x-1), \tag{2.23}$$

where $\delta(x)$ is Dirac delta function. The wave generated by such an idealized 'ship' is also Dirac delta function, i.e. an infinitely high and narrow pulse. The bow wave pulse, after reflection at the channel sidewall, is completely absorbed by the stern, so that there is no wave disturbance downstream and the wave resistance is zero. The wave characteristic lines form a closed rhombus as sketched in figure 5. This shape is typical of channel superconductivity and may be called a diamond wave pattern.

By contrast, if there were no channel sidewalls, both the bow and stern wave pulses would extend downstream to infinity and the wave resistance would be infinitely large within the linear theory, as can be seen by evaluating (2.13). But even a more realistic

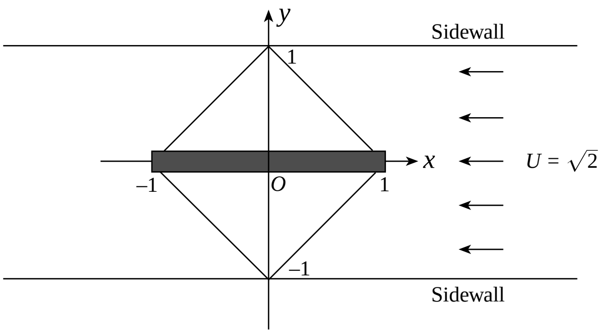

FIGURE 5. A superconductive rectangular ship generating a diamond pulse-wave pattern.

nonlinear analysis would predict a very high wave resistance for such a blunt-ended hull form. The analogy extends to gas dynamics and this is the well-known reason why supersonic aeroplanes normally have sharp ends rather than a blunt or even rounded head or tail.

3. Nonlinear analysis

Now we seek a more accurate nonlinear solution of the same problem. It goes beyond the preceding linear analysis by including effects of weak nonlinearity and dispersion. As shown by Mei (1976) and others, a stationary Kadomtsev–Petviashivili equation, or a stationary 2-D KdV equation,

$$(1-U^2)\varphi_{xx}+\varphi_{yy}+3U\varphi_x\varphi_{xx}+\tfrac{1}{3}U^2\varphi_{xxxx}=0 \quad (-\infty<x<\infty,\quad -w/2<y<w/2), \tag{3.1}$$

is obtained with the same boundary conditions (2.2) and (2.3) at the ship location and on the channel sidewalls, respectively. All variables have the same meanings as in the last section and are non-dimensionalized in the same way, see (2.4) etc.

Similarly to the linear analysis, we look for a no-wave solution in the supercritical range $U>1$. We know that the KP equation has a permanent single-soliton solution (Drazin & Johnson 1989). If we write this solution in the ξ, η framework for our purpose in the following form:

$$\varphi=F(\xi) \text{ or } \varphi=G(\eta), \tag{3.2}$$

where

$$\xi=x+y\cot\alpha,\quad \eta=x-y\cot\alpha, \tag{3.3}$$

then we have from (3.1),

$$(1-U^2+\cot^2\alpha)F''+3UF'F''+\tfrac{1}{3}U^2F''''=0. \tag{3.4}$$

The same equation holds for $G(\eta)$. If only the soliton solution is permitted, i.e. $F'(\xi)\to 0$ as $|\xi|\to\infty$, then the above equation can be integrated three times to yield its soliton solution. But it is more easily verified by direct substitution that

$$F'(\xi)=A\,\text{sech}^2(k\xi) \tag{3.5}$$

is a solution, provided

$$A=\frac{U^2-1-\cot^2\alpha}{U},\quad k=\frac{[3(U^2-1-\cot^2\alpha)]^{1/2}}{2U}. \tag{3.6}$$

Analogously, we have the same result for $G'(\eta)$, i.e.

$$G'(\eta) = A\ \mathrm{sech}^2(k\eta), \tag{3.7}$$

with (3.6). Physically, A is here the amplitude of longitudinal perturbation velocity and the amplitude of wave elevation is approximately UA based on the linear expression (2.8). The first equation of (3.6) yields

$$\tan\alpha = 1/(U^2 - 1 - AU)^{1/2}, \tag{3.8}$$

so we see that the slope of the characteristic lines now depends on the amplitude of the soliton but includes the linear-theory relation as the zero-amplitude limiting case. The soliton propagates normal to its crest line at a near-critical speed $U\sin\alpha = U^{1/2}/(U-A)^{1/2} \approx 1^+$ such that its induced speed in the direction of motion of the ship is again exactly equal to ship speed U as in the linear solution.

Inspired by the linear solution, we hope to find a hull form whose forebody would generate a solitary wave and whose afterbody would perfectly absorb the reflection of the bow wave from the channel sidewall. The solitary bow wave obliquely incident on a sidewall and its reflection are together equivalent to the oblique interaction of two identical solitons in water of infinite width. Such a general two-soliton solution of the KP (2-D KdV) equation is obtained, for example, by using the bilinear form given by Drazin & Johnson (1989, p. 124). So we can exploit it to determine the shape of the ship inversely. We derive a two-soliton solution of our KP equation (3.1) by means of a transformation, see the Appendix for details.

Mei (1976) gave the single-soliton solution of (3.1) for a so-called half-body comprising a long sharp nose protruding forward to $x \to \infty$ and a parallel body of constant cross-section extending aft to $x \to -\infty$ in infinitely wide shallow water. The wave resistance was given as well, see equation (6.11) in Mei (1976), and is non-zero for a body of finite thickness. It is easy to imagine that if we now add channel sidewalls suitably, similar to the linear analysis, the reflected soliton will close the half-body at $x \to -\infty$ such that no wave will be left in the wake.

Assume that the bow soliton is generated around a point B $(x_o, 0)$ in the forebody and that its reflection is centred around a point S $(-x_o, 0)$ in the afterbody, as sketched in figure 6. Let C $(0, w/2)$ be the centre point of the intersection of two identical solitons on the line $y = w/2$ such that the boundary condition on a channel sidewall (2.3) is satisfied. The two-soliton solution is given by (A 2) in the Appendix. Generally, by substituting φ of (A 2) into (2.2), we can obtain the zero-wave-resistance ship hull $S(x)$ for arbitrary channel width w. We give complete results in the Appendix, and it will be seen that the ship hull $S(x)$ is now fore-and-aft symmetric, i.e. $S(x)$ is an even function. For simplicity we assume here that the points B and S are sufficiently far from the centre point C, compared to the typical solitary wavelength $2\pi/k$, i.e. $kx_o \sin\alpha \gg 1$. Then the solution along the ship location is an asymptotic expression of the two-soliton solution, namely two isolated single solitons like (3.5) and (3.7),

$$\varphi_\xi = F(\xi - x_o) = A\ \mathrm{sech}^2 k(\xi - x_o) \quad (x > 0,\ \ y = +0), \tag{3.9}$$

$$\varphi_\eta = G(\eta + x_o) = A\ \mathrm{sech}^2 k(\eta + x_o) \quad (x < 0,\ \ y = +0). \tag{3.10}$$

Here the important soliton location parameter x_o is expressed as

$$x_o = -\tfrac{1}{2}\lambda + \tfrac{1}{2}w\cot\alpha, \tag{3.11}$$

where the nonlinear phase shift λ in the two-soliton interaction is found in the

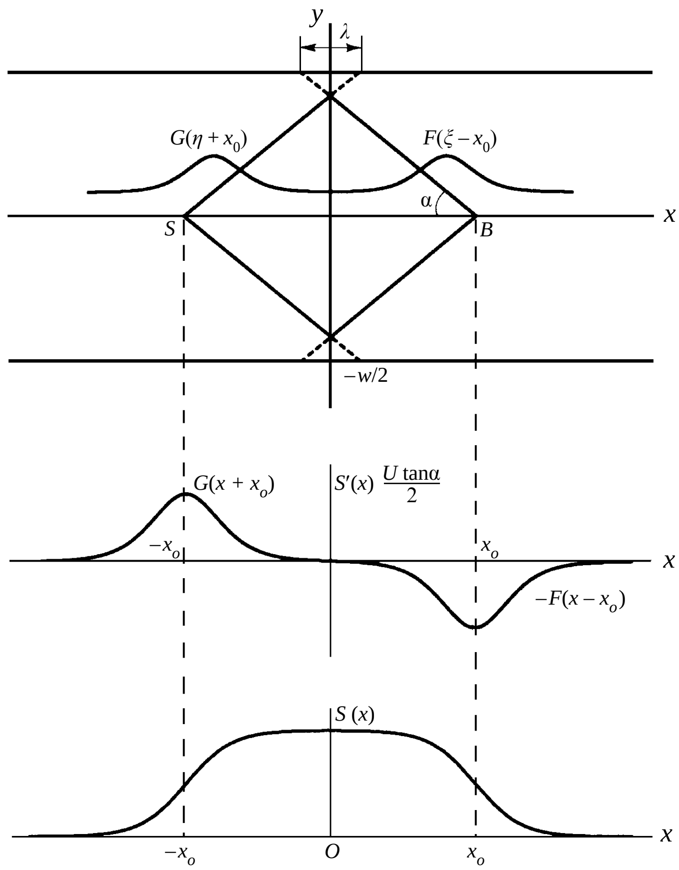

FIGURE 6. Schematic of zero-wave-resistance solution in nonlinear theory with a diamond-soliton wave pattern

Appendix to be

$$\lambda = \frac{U}{(3AU)^{1/2}} \log \frac{U^2 - 1 - AU}{U^2 - 1 - 4AU}. \tag{3.12}$$

We now derive the ship cross-sectional area from the hull boundary condition (2.2). Since

$$\varphi_y|_{y=+0} = \begin{cases} +\varphi_\xi \cot\alpha & (x > 0) \\ -\varphi_\eta \cot\alpha\varphi_\eta & (x < 0), \end{cases}$$

substituting it into the hull boundary condition (2.2) and using the expressions (3.9) and (3.10) yields

$$\frac{\mathrm{d}S(x)}{\mathrm{d}x} = -\frac{2}{U}\frac{\partial\varphi}{\partial y}\bigg|_{y=0^+} = \begin{cases} -\dfrac{2\cot\alpha}{U}\varphi_\xi = -\dfrac{2\cot\alpha}{U}F(x - x_o) & (x > 0) \\ +\dfrac{2\cot\alpha}{U}\varphi_\eta = +\dfrac{2\cot\alpha}{U}G(x + x_o) & (x < 0) \end{cases}$$

$$= -\mathrm{sgn}(x)\frac{2\cot\alpha}{U}A\ \mathrm{sech}^2[k(x - \mathrm{sgn}(x)x_o)]. \tag{3.13}$$

Integrating it once, taking advantage of the approximation $kx_o \gg 1$, yields

$$S(x) = S_o\{1 - \operatorname{sgn}(x)\tanh[k(x - \operatorname{sgn}(x)x_o)]\}/2, \tag{3.14}$$

where S_o is the midship sectional area of the hull,

$$S_o = \frac{4A\cot\alpha}{Uk}, \tag{3.15}$$

with arbitrary amplitude within the limits $0 < A < (U^2 - 1)/U$ and k and $\tan\alpha$ written inversely as functions of A from (3.6)

$$k = \frac{(3AU)^{1/2}}{2U}, \quad \tan\alpha = 1/(U^2 - 1 - AU)^{1/2}. \tag{3.16}$$

The above solution is sketched in figure 6.

Since $\varphi_x(x)$ according to (3.9) and (3.10) is an even function while $S'(x)$ according to (3.13) is an odd function, the integrated wave resistance according to (2.7) is automatically zero. The same holds for the general solution in the Appendix. This completes the desired nonlinear superconductive solution. The wave crest lines again form a rhombus, similar to the linear example in figure 5. But the nonlinear diamond pattern consists of smooth solitary waves rather than linear pulse waves, and the phase shift λ given in (3.12) lets the waves extend slightly beyond the transverse corners of the rhombus. This soliton pattern allows a slender ship with cross-sectional area distribution given by (3.14) to move at a supercritical speed without wave resistance, see figure 6.

We take the ship length $l = 4x_o$ approximately and calculate the ship displacement as

$$V = \int_{-2x_o}^{2x_o} S(x)\mathrm{d}x \approx 2x_o\frac{4A\cot\alpha}{Uk} = 2x_oS_o.$$

Then the longitudinal prismatic coefficient defined as $C_P = V/(S_ol)$, being one of the principal parameters of ship hull geometry, is found to be 0.5, which is a value within the conventional design range for fast inland or coastal ships.

We now discuss conditions for the existence of such diamond solitons. Using (3.16) and solving a quadratic equation, the amplitude A and wavenumber k of the soliton can be expressed directly in terms of midship sectional area S_o,

$$A = \frac{(U^2-1) \pm ((U^2-1)^2 - \frac{3}{16}U^2S_o^2)^{1/2}}{2U}, \quad k = \frac{3}{16}S_o\tan\alpha, \tag{3.17}$$

where

$$\tan\alpha = \frac{\sqrt{2}}{((U^2-1) \mp ((U^2-1)^2 - \frac{3}{16}U^2S_o^2)^{1/2})^{1/2}}. \tag{3.18}$$

Here only the lower choice of the signs is realistic since $A \to 0$ as $S_o \to 0$.

Note that nonlinearity puts a lower limit on supercritical speed, $U = (1 + 3S_o^2/16)^{1/2} + \sqrt{3}S_o/8$, at which superconductivity is possible. If we define a slenderness parameter $\epsilon = S_o^{1/2}/x_o \ll 1$, then $kx_o \approx (3/16\epsilon)S_o^{3/2}\tan\alpha$. So the condition $kx_o \gg 1$ implies $\epsilon \ll (3/16)S_o^{3/2}\tan\alpha$.

Besides the above condition, there are two other tacit prerequisites, namely no upstream soliton and no Mach stem. First, when an upstream soliton is generated, the problem becomes unsteady (e.g. see Wu 1987), which contradicts our stationarity assumption. But if U exceeds a certain value depending mainly on the ratio of the cross-sectional areas of ship and channel, for example $U > 1.25$ for the case reported in the Introduction, the flow will always be stationary. Second, it is known that if the incidence angle ψ, here $\psi = \pi/2 - \alpha$, is smaller than a critical value (in our version, it is arctan $(3AU)^{1/2}$, see the Appendix) depending on the soliton amplitude, reflection by the sidewall will generate a third wave, the so-called Mach stem (Miles 1977*b*; Melville 1980). So $\psi = \pi/2 - \alpha$ should be larger than this critical value to ensure the existence of a diamond soliton symmetric with respect to the midship section. This puts another lower bound on the superconductive speed. Pedersen (1988), based on his numerical results, pointed out that Mach reflection is also the cause of generation of the unsteady upstream solitons, which would imply that the two tacit prerequisites above are one and the same thing. But this proposition has not been proved analytically. Anyway, even with a Mach stem, a stationary three-soliton wave pattern (Miles 1977*b*) is still possible in a suitably diverging channel such that the triple point moves parallel to the ship's track. The corresponding hull form would be neither fore-and-aft asymmetric nor closed. Since the Mach stem becomes steadily wider in the diverging channel and carries mass and energy with it, the wave resistance cannot vanish but may be small enough to be interesting.

Finally, it is a pleasure to observe that a hexagonal wave pattern is a typical permanent wave pattern in shallow water, as was remarkably demonstrated in experiments and well described with the KP equation by Hammack, Scheffner & Segur (1989) and Hammack *et al.* (1995). Such genuinely two-dimensional, periodic wave patterns were also reported earlier by Peregrine (1985). Comparing the hexagonal free wave pattern with the superconductive rhombic ship wave pattern, we can imagine that they are related. If we extend the idea to the case of periodic waves, we can get a superconductive 'ship' periodically repeating from $x = -\infty$ to $x = \infty$ adapted to the given channel width.

4. Concluding remarks

We have shown by linear shallow-water theory, and confirmed by nonlinear analysis, that slender ships can be designed to run at supercritical speeds on the centreline of a straight rectangular channel without experiencing wave resistance. This phenomenon may be aptly called channel superconductivity. Although we have based our nonlinear analysis on the KP equation, the physical idea can be described by different equations to their different degrees of accuracy. In another paper, see Chen & Sharma (1996), we have used the more general Boussinesq equations but limited to weak-interaction soliton solutions of Miles (1977*a*).

While unidirectional linear or weakly nonlinear shallow-water waves can propagate only at the critical or a near-critical speed, the localized superconductive wave pattern can move at any speed that is larger or even much larger than the critical speed. This observation is not entirely new for water waves but may be significant if it could be extended to other kinds of waves, e.g. light waves.

From a practical point of view it may be unrealistic to assume the existence of a long uniform canal or river with smooth vertical sidewalls. So we have also investigated the possibility of wave resistance reduction by optimal wave interference between the twin hulls of a catamaran, which is mathematically equivalent to a single hull moving parallel to an ideally reflecting sidewall. It is a classical option for high-speed ships. By exploiting the new ideas presented here it seems that up to 50% reduction of wave resistance is possible for a suitable configuration of symmetric hulls and theoretically up to 100% for appropriately cambered hulls. As regards absolute speed, a depth

Froude number $U = 1.5$ in inland or coastal waters of 10–30 m depth corresponds to ship speeds ranging from 53 to 93 km h^{-1}, i.e. 29–50 knots, which are well within the range of present technical feasibility and economic interest.

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Appendix. Two-soliton solution of the KP (2-D KdV) equation and the derived zero-wave-resistance ship hull

A general two-soliton solution has been given in Drazin & Johnson (1989) for the KP equation in terms of the bilinear form of Hirota's method. The solution was originally derived by Satsuma (1976). Because our KP equation has a different form, it is necessary to transform the results for our purpose.

After a transformation

$$t \to t, \quad x \to x + \frac{3(U^2-1)}{U^2}\,t, \quad y \to Uy, \quad u = -\frac{3}{2U}\varphi_x, \tag{A 1}$$

and setting $\partial/\partial t = 0$, the 2-D KdV (KP) equation in Q5.30 of Drazin & Johnson (1989) becomes our physical equation (3.1). So the two-identical-soliton solution of (3.1) is obtained for our purpose in the form,

$$\varphi_x = -\frac{2U}{3}u = \frac{4U}{3}\frac{\partial^2}{\partial x^2}\log f, \quad \text{or} \quad \varphi = \frac{4U}{3}\frac{\partial}{\partial x}\log f, \tag{A 2}$$

where, in terms of variables that are used in §3,

$$f = 1 + \exp(2k\xi) + \exp(2k\eta) + A_2\exp(2k\xi)\exp(2k\eta), \tag{A 3}$$

with $\xi = x + y\cot\alpha - x_1 + \delta_o$, $\eta = x - y\cot\alpha + x_1 + \delta_o$, where δ_o is left to be selected such that the solution will be symmetric about the y-axis, and

$$A_2 = \frac{U^2 - 1 - AU}{U^2 - 1 - 4AU}, \tag{A 4}$$

involving the functions k and $\tan\alpha$ of A as given in (3.16), and A is the amplitude of its asymptotic single soliton (3.9) or (3.10). The centre point C ($x = 0, y = w/2$) of the intersection of two identical solitons has been fixed on the channel sidewall. Hence the half phase shift is determined as

$$\delta_o = -\frac{1}{4k}\log A_2, \tag{A 5}$$

and the phase shift λ in the x-direction, defined in (3.11), as

$$\lambda = -2\delta_o. \tag{A 6}$$

The asymptotic solitons (3.9) and (3.10) can be obtained from (A 2) by setting $x_1 = x_o - \delta_o$ in ξ or η while $\eta \to +\infty$ and ξ remains finite, or $\xi \to -\infty$ and η remains finite, respectively.

We see that the two-identical-soliton solution (regular reflection) is possible if

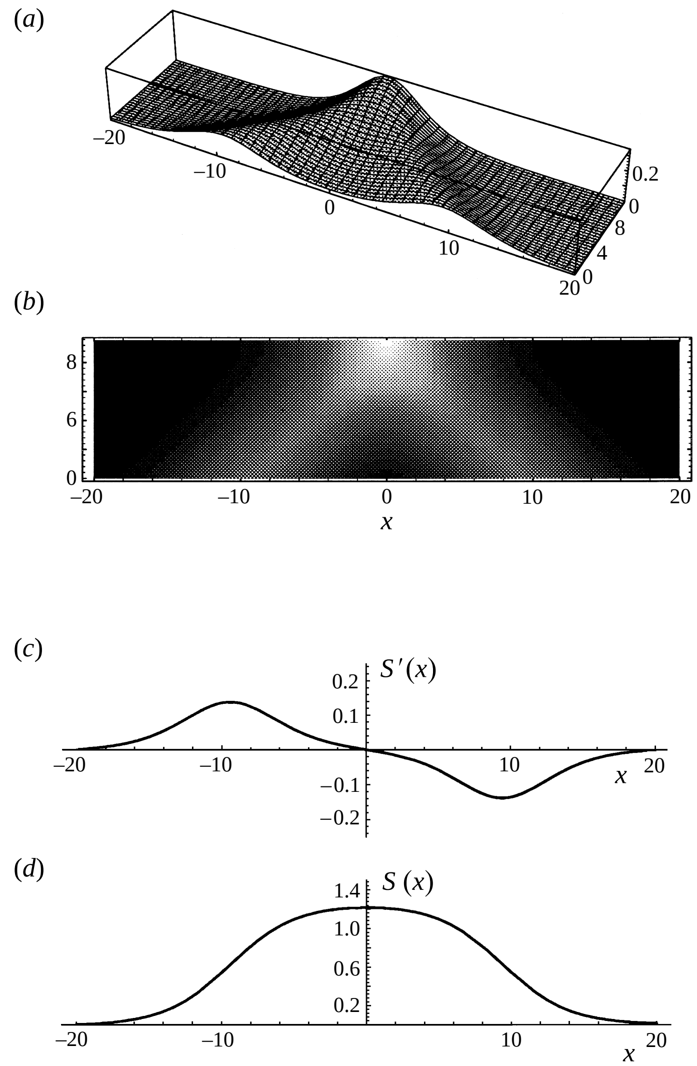

FIGURE 7. The zero-wave-resistance ship hull and its associated wave pattern in the case of $U = 1.5$, $A = 0.1$ and $x_1 = 10$. (*a*) Three-dimensional plot of the wave pattern; (*b*) density plot of the wave pattern; (*c*) slope of sectional area curve $S'(x)$; (*d*) cross-sectional area curve of ship hull $S(x)$.

$A_2 > 0$. It yields a critical value of the incidence angle $\psi = \pi/2 - \alpha$:

$$\tan^2\psi > 3AU, \quad \text{i.e.} \quad A < \frac{U^2-1}{4U}, \tag{A 7}$$

which is equivalent to (1.10) of Miles (1977*b*) as A is small. When ψ is less than this value, the solution becomes singular, in the sense that regular incoming waves with sech^2 profiles yield singular outgoing waves with -cosech^2 profile (Miles 1977*a*). Physically, the phenomenon of Mach reflection occurs (see Miles 1977*b*).

Finally, we derive the zero-wave-resistance ship hull $S(x)$ from the two-soliton solution (A 2). Substituting φ of (A 2) into (2.2), integrating once with respect to x

and after straightforward calculation, one obtains

$$S(x) = -\frac{8}{3}\frac{\partial}{\partial y}\log f|_{y=+0} = \frac{16}{3}\,\frac{k\cot\alpha\,\sinh(2kx_1)}{\cosh(2kx_1) + A_2^{1/2}\cosh(2kx)}, \tag{A 8}$$

where A_2 is given in (A 4), k and $\tan\alpha$ in (3.16), and $w = 2x_1\tan\alpha$ obtained from the boundary condition (2.3) on the channel sidewalls, i.e. $f_y = 0$ at $y = \pm w/2$. So only A and w (or x_1) are free to be selected for designing the ship hull, where x_1 can be arbitrary and A is restricted by (A 7). $S(x)$ is an even function. This means the derived zero-wave-resistance ship hull is fore-and-aft symmetric without considering the detailed shape of cross-section. For $kx_1 \gg 1$, we can get the same asymptotic result as in (3.14) with $x_1 = x_o - \delta_o$.

As an example, in figure 7 we show a zero-wave-resistance solution of a ship hull and its associated wave pattern for chosen values $U = 1.5$, $A = 0.1$, $x_1 = 10$, and derived values $w = 19.0693$, $S(0) = 1.214$.

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